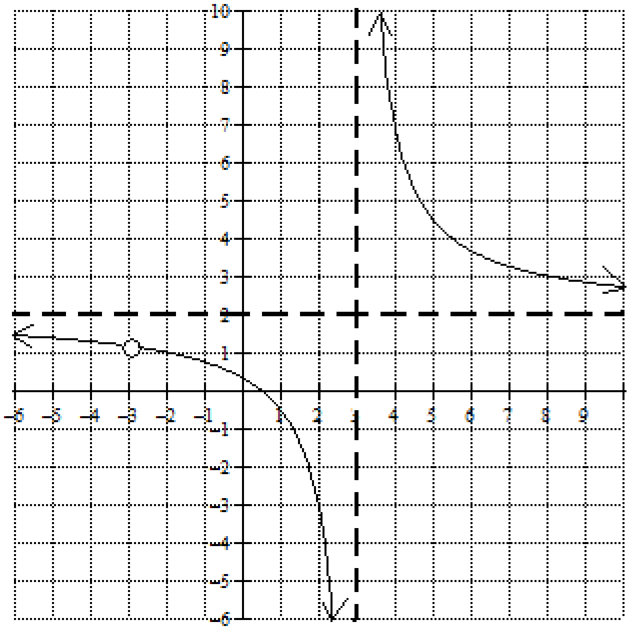
Lesson #2- Understanding the Limit
A Numerical and Graphical Approach

The equation of the function graphed to the right is $f(x)=\dfrac{2x^2+5x-3}{x^2-9}$. The coordinates of the hole in the graph are $\left(-3,\frac{7}{6}\right)$.



Pre-calculus Statements	Calculus Limit Notation
As $x\to-\infty$, the graph of $f(x)\to$ __________.	
As $x\to\infty$, the graph of $f(x)\to$ __________.	
As $x\to -3$ from the left, the graph of $f(x)\to$ __________.	
As $x\to -3$ from the right, the graph of $f(x)\to$ __________.	
As $x\to 3$ from the left, the graph of $f(x)\to$ __________.	
As $x\to 3$ from the right, the graph of $f(x)\to$ __________.	

Based on what you have just seen, how might you informally define what the value of a limit represents in terms of the graph?

A Numerical Analysis of Limits

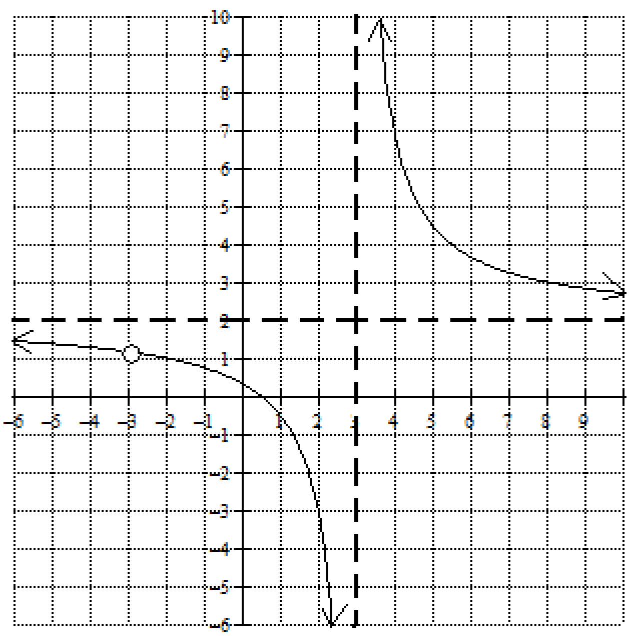
- Now let's consider the function $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 - 9}$
- Complete the table to below to perform a numerical analysis of the function as $x \to -\infty$ and ∞.

x	−1000	−500	−100	−50	50	100	500	1000
$f(x)$								

Based on the values in the table, what are the values of

$$\lim_{x \to -\infty} \frac{2x^2 + 5x - 3}{x^2 - 9} \qquad\qquad \lim_{x \to \infty} \frac{2x^2 + 5x - 3}{x^2 - 9}$$

How is the numerical analysis above related in the graph of the function pictured below?



A Numerical Analysis of Limits

- Now let's consider the function $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 - 9}$

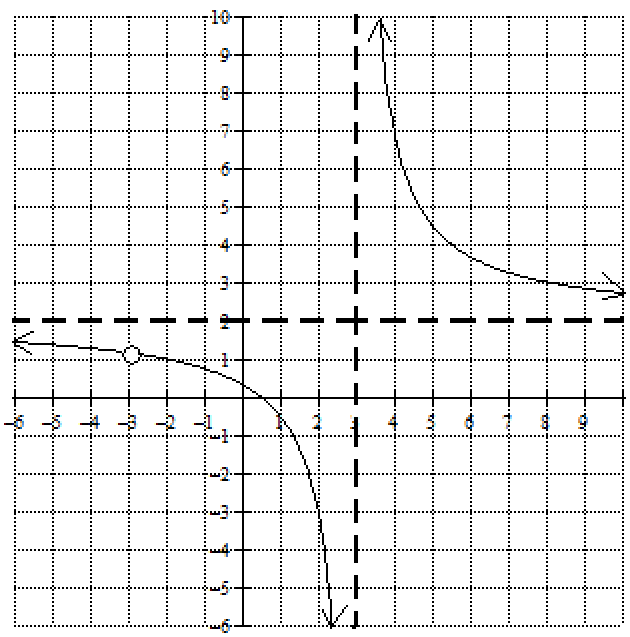
- Complete the table to below to perform a numerical analysis of the function as $x \to -3$ from the left and from the right.

x	−3.75	−3.1	−3.01	−3.001	−2.999	−2.99	−2.9	−2.75
$f(x)$								

Based on the values in the table, what are the values of

$$\lim_{x \to -3^-} \frac{2x^2 + 5x - 3}{x^2 - 9} \qquad\qquad \lim_{x \to -3^+} \frac{2x^2 + 5x - 3}{x^2 - 9}$$

How is the numerical analysis above related in the graph of the function pictured below?



A Numerical Analysis of Limits

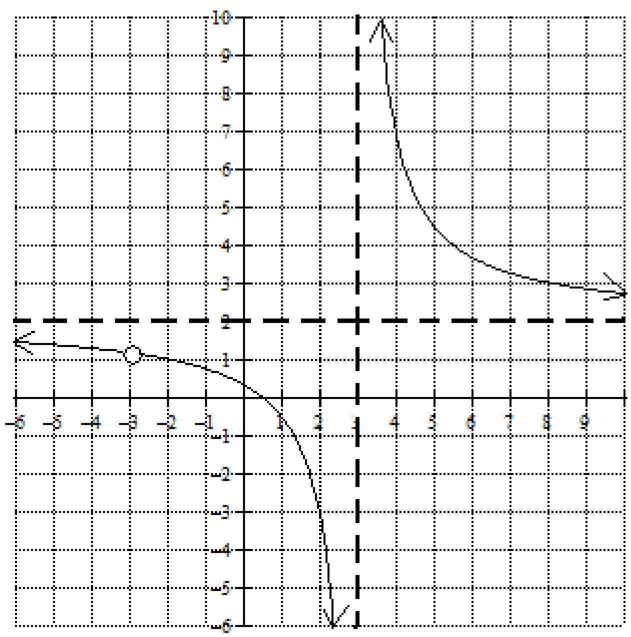
- Now let's consider the function $f(x)=\dfrac{2x^2+5x-3}{x^2-9}$
- Complete the table to below to perform a numerical analysis of the function as $x \to 3$ from the left and from the right.

x	2.75	2.9	2.99	2.999	3.001	3.01	3.1	3.75
f(*x*)								

Based on the values in the table, what are the values of

$$\lim_{x\to 3^-}\frac{2x^2+5x-3}{x^2-9} \qquad\qquad \lim_{x\to 3^+}\frac{2x^2+5x-3}{x^2-9}$$

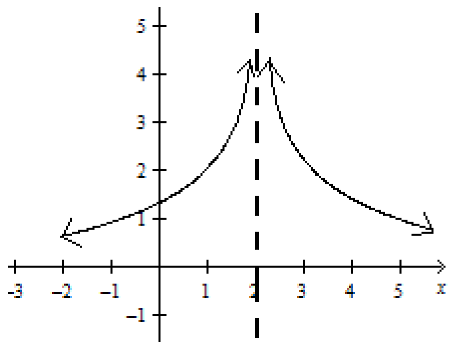
How is the numerical analysis above related in the graph of the function pictured below?



Limit Existence Theorem

Limits That Do Not Exist

Example #1



Find each of the following from the graph.

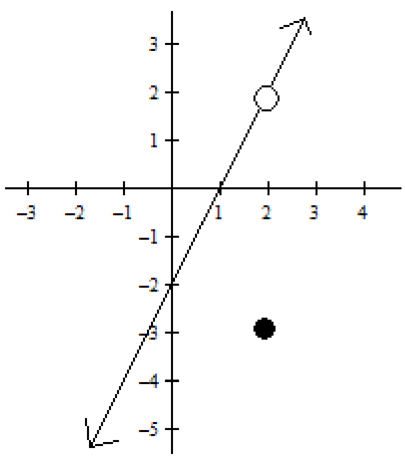
a) $\lim\limits_{x \to 2^-} f(x) =$

b) $\lim\limits_{x \to 2^+} f(x) =$

c) $f(2) =$

d) Does $\lim\limits_{x \to 2} f(x)$ exist or not? Why or why not?

Example #2



Find each of the following from the graph.

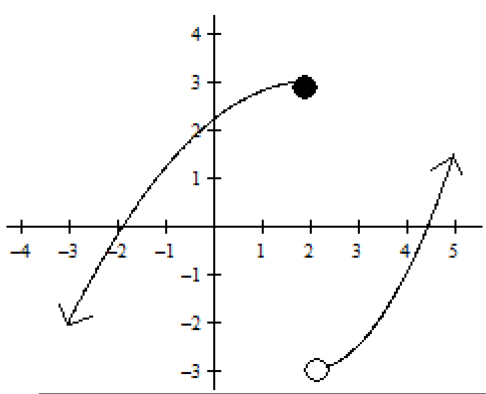
a) $\lim\limits_{x \to 2^-} f(x) =$

b) $\lim\limits_{x \to 2^+} f(x) =$

c) $f(2) =$

d) Does $\lim\limits_{x \to 2} f(x)$ exist or not? Why or why not?

Example #3



Find each of the following from the graph.

a) $\lim\limits_{x \to 2^-} f(x) =$

b) $\lim\limits_{x \to 2^+} f(x) =$

c) $f(2) =$

d) Does $\lim\limits_{x \to 2} f(x)$ exist or not? Why or why not?

Based on what you have seen so far, does $f(a)$ have to be defined in order for the $\lim_{x \to a} f(x)$ to exist? Draw and explain two different graphs to justify your reasoning. In both graphs, $f(a)$ should be undefined but in one graph, the limit should exist while in the second one, it should not exist.

Below is a table of values of an exponential function. Use the table to find the limits that follow.

x	-9	-5	-3	-1	1	3	9
$f(x)$	513	33	9	3	1.5	1.125	1.002

a) $\lim_{x \to -\infty} f(x)$

b) $\lim_{x \to -3} f(x)$

c) $\lim_{x \to 1} f(x)$

d) $\lim_{x \to \infty} f(x)$

Below is a table of values of a rational function. Use the table to find the limits that follow.

x	-1000	-1.001	-1	-0.999	0	1.999	2	2.001	1000
$F(x)$	1.002	2001	Undefined	-1999	-1	0.333	Undefined	0.334	0.998

a) $\lim_{x \to -\infty} f(x)$

b) $\lim_{x \to -1^-} f(x)$

c) $\lim_{x \to -1^+} f(x)$

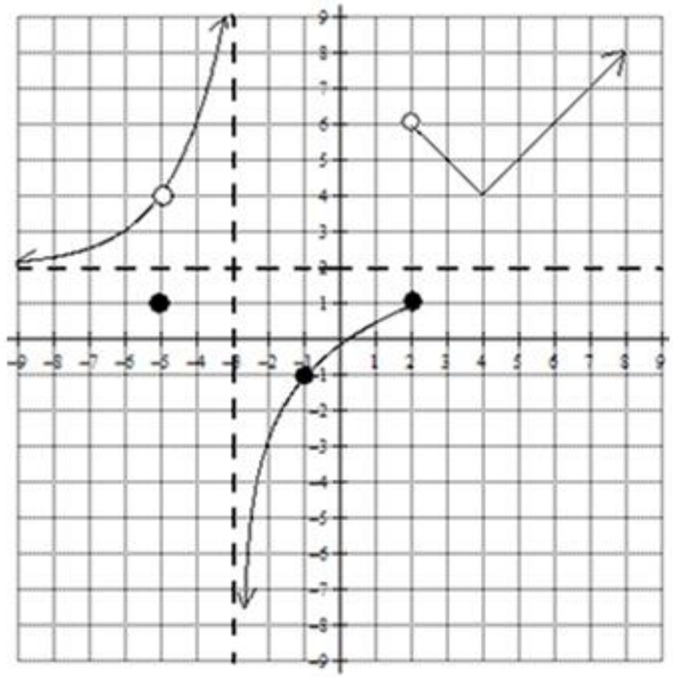
d) $\lim_{x \to 2} f(x)$

e) $\lim_{x \to \infty} f(x)$

f) $\lim_{x \to -1} f(x)$

A Graphical Analysis of Limits

- Consider the graph of the function, f(x), graphed below.



A Graphical Analysis of Limits

Using the graph, find the value of each of the following limits. If a limit does not exist, state why.

a) $\lim\limits_{x \to -3^-} f(x)$

b) $\lim\limits_{x \to -5} f(x)$

c) $\lim\limits_{x \to -1} f(x)$

c) $\lim\limits_{x \to -3} f(x)$

d) $\lim\limits_{x \to 2^-} f(x)$

e) $\lim\limits_{x \to 2^+} f(x)$

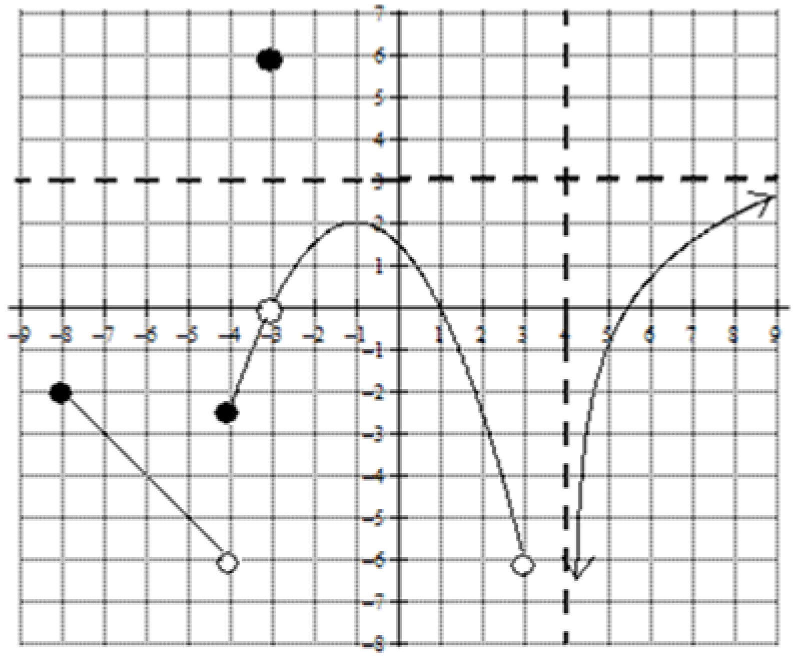
f) $\lim\limits_{x \to 2} f(x)$

g) $\lim\limits_{x \to -\infty} f(x)$

h) $\lim\limits_{x \to \infty} f(x)$

A Graphical Analysis of Limits

Consider the graph of the function, g(x), graphed below.



A Graphical Analysis of Limits

Find the value of each of the following limits using the graph of g(x). If a limit does not exist, state why.

a) $\lim\limits_{x \to -3^-} g(x)$ b) $\lim\limits_{x \to -6} g(x)$ c) $\lim\limits_{x \to -1^+} g(x)$

d) $\lim\limits_{x \to -3^+} g(x)$ e) $\lim\limits_{x \to 4^-} g(x)$ f) $\lim\limits_{x \to 4^+} g(x)$

g) $\lim\limits_{x \to 4} g(x)$ h) $\lim\limits_{x \to -4^+} g(x)$ i) $\lim\limits_{x \to -4^-} g(x)$

Name____**Date**____________________**Class**__________

Lesson #1 and 2 Homework

Below are tables of values for given types of functions. For each table, the type of function represented by the table is given. Use your knowledge of the numerical behavior of each type of function to find the indicated limits. For limits that do not exist, write D.N.E.

1. Exponential Function

x	–7	–4	–1	2	5	8	11
$H(x)$	–125	–13	1	2.75	2.969	2.996	2.999

a) $\lim_{x\to-\infty} H(x) =$ b) $\lim_{x\to-1} H(x) =$ c) $\lim_{x\to\infty} H(x) =$

2. Rational Function

x	–1000	–2.001	–2	–1.999	0.999	1	1.001	1000
$G(x)$	0.998	0.333	Undefined	0.333	–1999	Undefined	2001	1.002

a) $\lim_{x\to-\infty} G(x) =$ b) $\lim_{x\to-2^-} G(x) =$ c) $\lim_{x\to-2^+} G(x) =$

d) $\lim_{x\to-2} G(x) =$ e) $\lim_{x\to1^-} G(x) =$ f) $\lim_{x\to1^+} G(x) =$

g) $\lim_{x\to1} G(x) =$ h) $\lim_{x\to\infty} G(x) =$

3. Rational Function

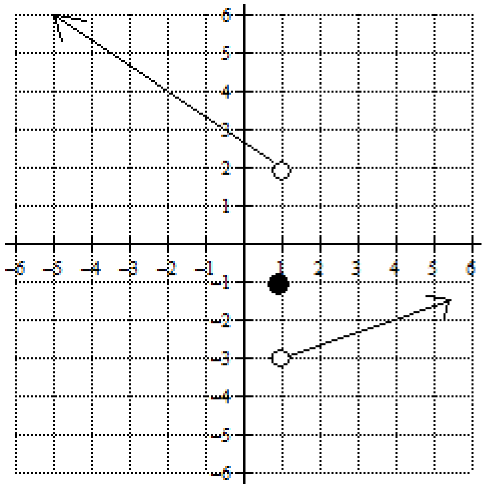
x	–10000	0.999	1	1.001	3.999	4	4.001	10000
$H(x)$	1.9999	–2.331	Undefined	–2.335	–12998	Undefined	13002	2.001

a) $\lim_{x\to\infty} H(x) =$ b) $\lim_{x\to1} H(x) =$ c) $\lim_{x\to4^+} H(x) =$

d) $\lim_{x\to4^-} G(x) =$ e) $\lim_{x\to4} G(x) =$

If they exist, determine the indicated values below each graph. For limits that do not exist, write D.N.E.

4. The graph of $h(x)$ is given.

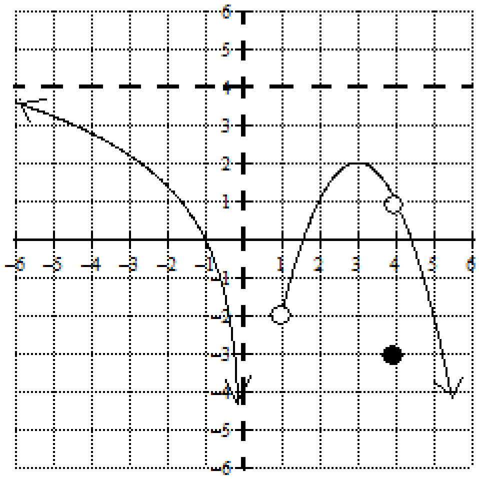


a) $\lim_{x\to 1^-} h(x) =$ b) $\lim_{x\to 1^+} h(x) =$

c) $\lim_{x\to 1} h(x) =$ d) $h(1) =$

e) $h(-2) =$ f) $\lim_{x\to -2} h(x) =$

5. The graph of $g(x)$ is given.

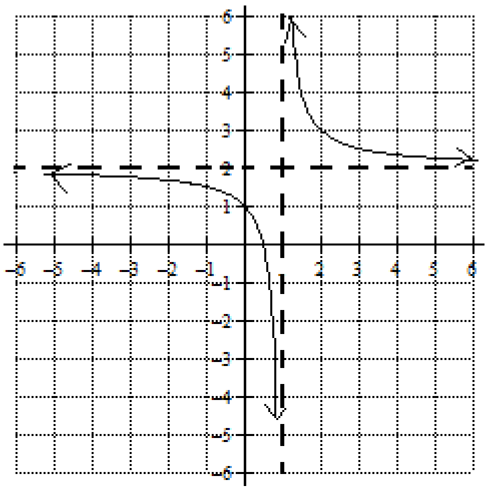


a) $\lim_{x\to 0^-} g(x) =$ b) $\lim_{x\to 1^+} g(x) =$

c) $\lim_{x\to -\infty} g(x) =$ d) $\lim_{x\to 4} g(x) =$

e) $g(4) =$ f) $\lim_{x\to 3} g(x) =$

6. The graph of $f(x)$ is given.

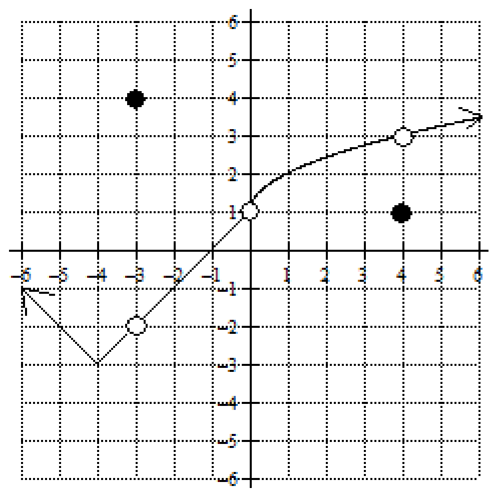


a) $\lim_{x\to 0} f(x) =$ b) $\lim_{x\to -\infty} f(x) =$

c) $\lim_{x\to \infty} f(x) =$ d) $\lim_{x\to 1^+} f(x) =$

e) $\lim_{x\to 1^-} f(x) =$ f) $\lim_{x\to 1} f(x) =$

7. The graph of $q(x)$ is given.

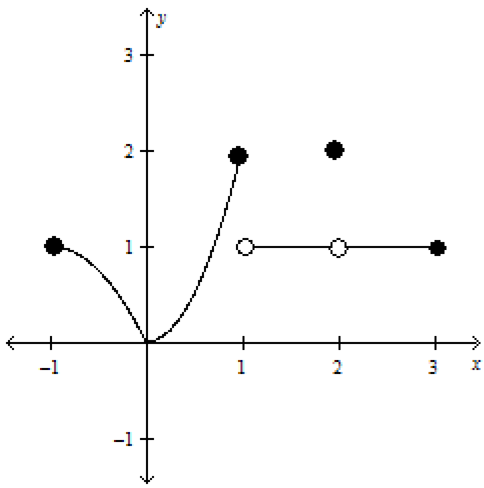


a) $\lim_{x\to 0} q(x) =$ b) $\lim_{x\to -3} q(x) =$

c) $\lim_{x\to 4} q(x) =$ d) $\lim_{x\to -4} q(x) =$

e) $q(-3) =$ f) $q(4) =$

Given the graph of the function, $g(x)$, below, determine if the statements are true or false. For statements that are false, explain why.



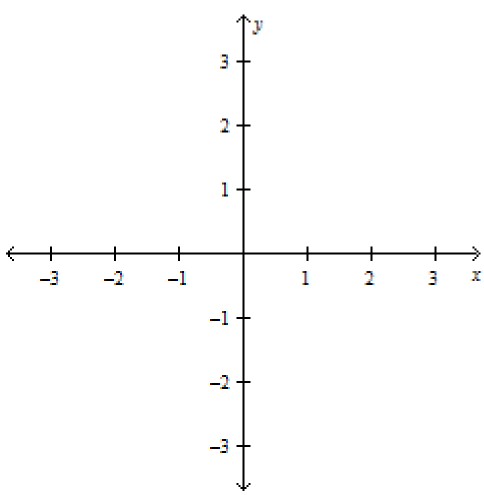
8. $\lim_{x \to 1} g(x) = 2$

9. $\lim_{x \to c} g(x)$ exists for every value of c on the interval (–1, 1).

10. $\lim_{x \to 2} g(x)$ does not exist.

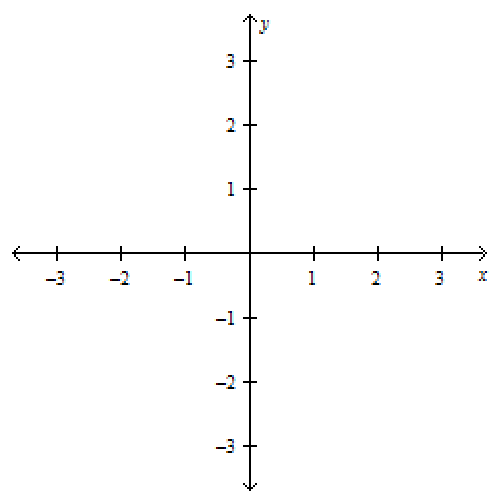
Sketch a graph of a function that fits the requirements described below.

11. $\lim_{x \to 1^-} f(x) = 3$ $\quad \lim_{x \to 1^+} f(x) = -1$ $\quad f(1) = 1$



12. $\lim_{x \to -2^-} f(x) = -\infty$ $\quad \lim_{x \to -2^+} f(x) = \infty$

$f(2)$ is undefined but $\lim_{x \to 2} f(x)$ exists.



13. In exercise 11, does $\lim_{x \to 1} f(x)$ exist? Explain why or why not.