Lesson #6- Limit-Based Continuity

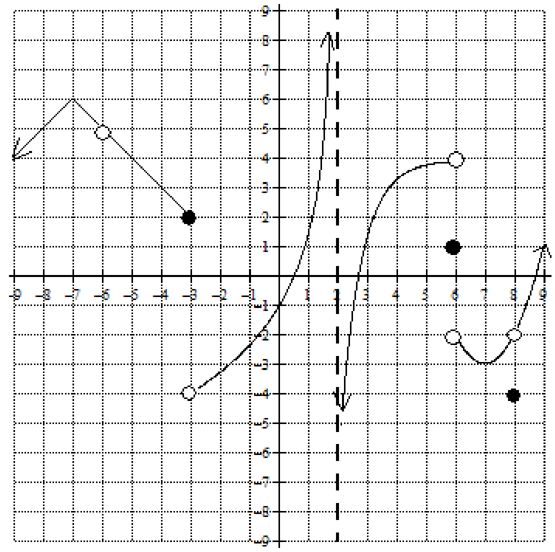
Graphical and Analytical Approaches

For the function graphed below, fill in the table with the given information. After filling in the table, write three pieces of information that must be true in order for a function, $G(x)$, to be continuous at $x = a$.

1.

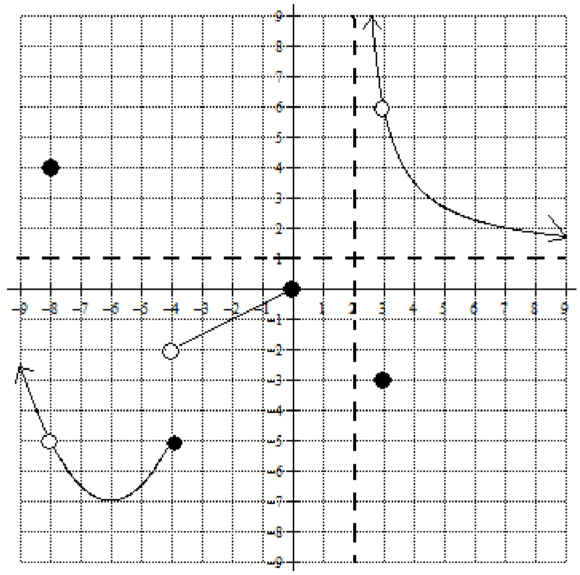
2.

3.



$x = a$	Is the function defined? If so, what is its value?	What is the value of $\lim_{x \to a^-} G(x)$?	What is the value of $\lim_{x \to a^+} G(x)$?	What is $\lim_{x \to a} G(x)$?	Is $G(x)$ continuous at $x = a$?
$x = -6$					
$x = -3$					
$x = 0$					
$x = 2$					
$x = 6$					
$x = 8$					

The graph of the function, $G(x)$, pictured to the right has several x – values at which the function is not continuous. For each of the following x – values, use the three part definition of continuity to determine if the function is continuous or not.



1. $x = -8$	2. $x = -6$	3. $x = -4$

Use the three part definition of continuity to determine if the given functions are continuous at the indicated values of *x*.

4. $f(x)=\begin{cases} -2\sqrt{x+6}, & x<-2 \\ 3x+2, & x=-2 \\ e^{x}+\cos(\pi x), & x>-2 \end{cases}$ at $x=-2$	5. $g(x)=\begin{cases} e^{x}\cos x, & x<\pi \\ e^{x}\tan\left(\frac{3x}{4}\right), & x\geq\pi \end{cases}$ at $x=\pi$

$$f(x)=\begin{cases} 2, & x\leq -1 \\ mx+k, & -1<x<3 \\ -2, & x\geq 3 \end{cases}$$

6. Consider the function, $f(x)$, to the right to answer the following questions.

   a. What two limits must equal in order for $f(x)$ to be continuous at $x = -1$?

   b. What two limits must equal in order for $f(x)$ to be continuous at $x = 3$?

   c. Determine the values of *m* and *k* so that the function is continuous everywhere.

7. Consider the function, $g(x)$, to the right to answer the following questions.

$$g(x)=\begin{cases} kx^2+m, & x<-2 \\ 4x+1, & -2\le x\le 3 \\ kx-m, & x>3 \end{cases}$$

a. What two limits must equal in order for $g(x)$ to be continuous at $x = -2$?

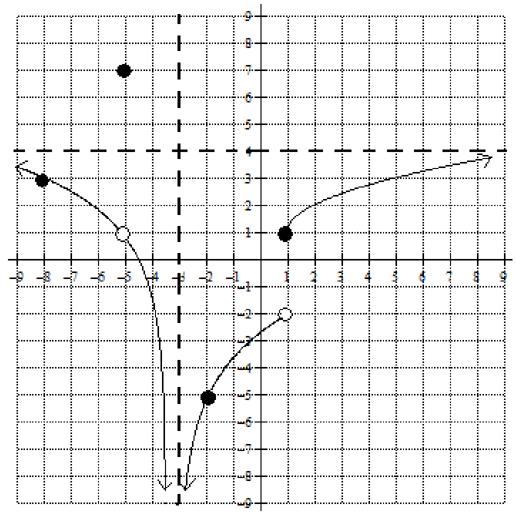
b. What two limits must equal in order for $g(x)$ to be continuous at $x = 3$?

c. Determine the values of m and k so that the function is continuous everywhere.

Name____Date___________________Class_________

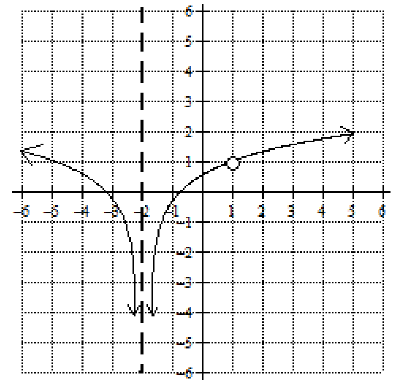
Lesson #6 Homework

For exercises 1 – 3, determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.

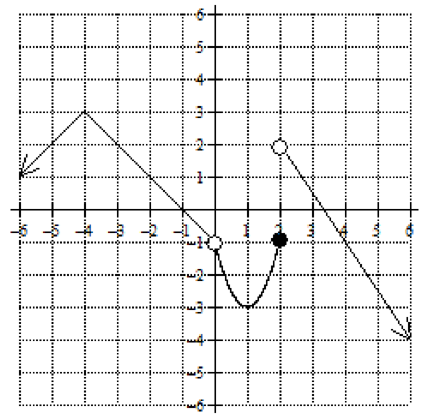


1. $x = -5$	2. $x = 1$	3. $x = -2$

4. Write a discussion of continuity for each of the function, $g(x)$, below. Be sure to include in your discussion where $g(x)$ is continuous and where $g(x)$ is discontinuous. For values at which the function is discontinuous, explain using the three part definition of continuity.



5. Use the three part definition of continuity to graphically justify why $p(x)$ is discontinuous at $x = 0$ and $x = 2$.



6. For what values of k and m is the function $g(x)$ everywhere continuous? Use limits to set up your work.

$$g(x) = \begin{cases} kx^2 + m, & x < -2 \\ e^{\ln(2x+3)}, & -2 \le x \le 3 \\ kx - m, & x > 3 \end{cases}$$

Find the value of *a* that makes each of the functions below everywhere continuous. Write the two limits that must be equal in order for the function to be continuous.

7. $f(x)=\begin{cases}4-x^2, & x<-1\\ ax^2-1, & x\geq -1\end{cases}$	8. $f(x)=\begin{cases}x^2+x+a, & x<2\\ ax^3-x^2, & x\geq 2\end{cases}$