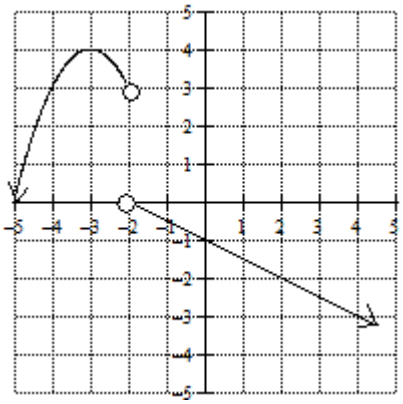
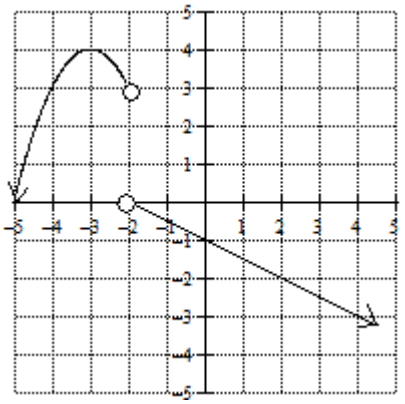
Lesson #7- Intermediate Value Theorem

As we study calculus, we will study several different theorems. The first theorem of investigation is the Intermediate Value Theorem. Together, let's write the theorem.

Intermediate Value Theorem

Now, investigate the graphs below to determine if the theorem is applicable for these functions on the specified intervals for the values given.

$f(x)=\begin{cases}-(x+3)^2+4, & x<-2\\ -\frac{1}{2}x-1, & x>-2\end{cases}$ 	$f(x)=\begin{cases}-(x+3)^2+4, & x<-2\\ -\frac{1}{2}x-1, & x>-2\end{cases}$ 
Is there a value of c on $[-5, 2]$ such that $f(c) = 2$?	Is there a value of c $[-1, 5]$ such that $f(c) = 2$?
Does the I.V.T. guarantee a value of c such that $f(c) = 2$ on the interval $[-5, 2]$? Why or why not?	Does the I.V.T. guarantee a value of c such that $f(c) = 2$ on the interval $[-1, 5]$? Why or why not?

What two conditions must be true to verify the applicability of the Intermediate Value Theorem?

1._____

2._____

For each of the following functions, determine if the I.V.T. is applicable or not and state why or why not. Then, if it is applicable, find the value of *c* guaranteed to exist by the theorem.

1. $f(x) = \frac{x-3}{x+2}$ on the interval [–1, 3] for $f(c) = \frac{2}{3}$	2. $f(x) = \frac{x-3}{x+2}$ on the interval [–4, 1] for $f(c) = \frac{2}{3}$
3. $p(x) = e^{x+2}\cos x$ on the interval [−2, 1] for $p(c) = 5$	

4. $f(x) = \frac{x}{x-2}$ on the interval [–1, 1] for $f(c) = -\frac{1}{2}$	5. $f(x) = -\left(\frac{1}{2}\right)^{-x+3} - 2$ on the interval [3, 5] for $f(c) = -4$

Name_____**Date**_____**Class**_____

Lesson #7 Homework

1. Determine, using the intermediate value theorem, if the function $F(x) = x^3 + 2x - 1$ has a zero on the interval [0, 1]. Justify your answer and find the indicated zero, if it exists.

2. Determine, using the intermediate value theorem, if the function $g(\theta) = \theta^2 - 2 - \cos\theta$ has a zero on the interval $[0, \pi]$. Justify your answer and find the indicated zero, if it exists.

For exercises 3 – 5, first, verify that the I.V.T. is applicable for the given function on the given interval. Then, if it is applicable, find the value of the indicated c, guaranteed by the theorem.

3. $f(x) = x^2 - 6x + 8$ Interval: [0, 3] $f(c) = 0$

4. $g(x) = x^3 - x^2 + x - 2$ Interval: [0, 3] $g(c) = 4$

5. $h(x) = \dfrac{x^2 + x}{x - 1}$ Interval: $\left[\frac{5}{2}, 4\right]$ $h(c) = 6$