## Lesson #9- AP Calculus Multiple Choice Practice
## Graphing Calculator NOT Permitted – 20 minutes

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1. $\lim\limits_{x \to 0} \dfrac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

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$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

2. The function $f$ is defined above. For what value of $k$, if any, is $f$ continuous at $x = 2$ ?

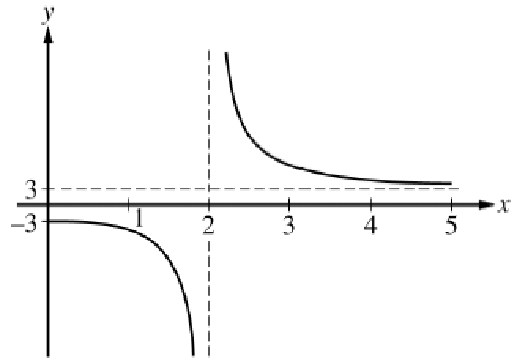
(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of $k$ will make $f$ continuous at $x = 2$.




3. The function $f$ is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of $f$. Which of the following could be the values of the constants $a$ and $b$ ?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

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4. If $f(x) = 3x^2 + 2x$, then $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ is…

(A) $6x + 2$
(B) $6x$
(C) 0
(D) nonexistent
(E) 2

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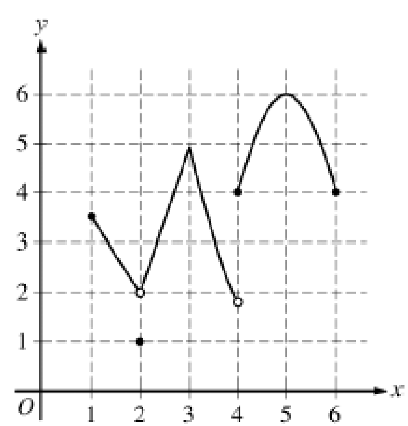
5.



Graph of $f$

The graph of the function $f$ is shown above. Which of the following statements is false?

(A) $\lim\limits_{x \to 2} f(x)$ exists.

(B) $\lim\limits_{x \to 3} f(x)$ exists.

(C) $\lim\limits_{x \to 4} f(x)$ exists.

(D) $\lim\limits_{x \to 5} f(x)$ exists.

(E) The function $f$ is continuous at $x = 3$.

$$g(x)=\begin{cases} \sin\frac{x\pi}{4}, & x<3 \\ x\sqrt{2}, & x=3 \\ \frac{x\sqrt{2}}{4x-6}, & x>3 \end{cases}$$

6. For the function above, which of the following would be the reason(s) why the function, $g(x)$, is not continuous at $x = 3$?

I. $g(3)$ is undefined.   II. $\lim_{x\to 3} g(x)$ does not exist.   III. $\lim_{x\to 3} g(x) \neq g(3)$.

(A) III only
(B) II only
(C) I and II only
(D) I only
(E) II and III only

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7. $\lim_{x\to 2^+} \frac{3-2x}{x-2}$ is…

(A) 0
(B) $\infty$
(C) $-\infty$
(D) 2
(E) ½

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8. Let $f$ be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

**AP**

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

(B) $f(3) = 15$

(C) $f$ attains a maximum on the open interval $(2, 4)$.

(D) $f(x) = 9$ has at least one solution in the open interval $(2, 4)$.

(E) $f(x)$ has at least one zero in the open interval $(2, 4)$.

**Calculus Free Response Practice #1**
**Calculator Permitted**

Consider the function $h(x)=\dfrac{-2x-\sin x}{x-1}$ to answer the following questions.

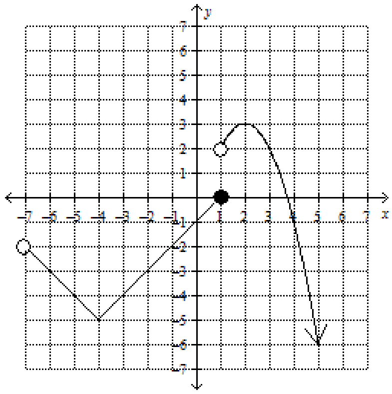
a. Find $\lim\limits_{x\to\frac{\pi}{2}}\left[h(x)\cdot(2x-2)\right]$. Show your analysis.

b. Identify the vertical asymptote(s), if any exist, of $h(x)$ and justify the existence by writing a limit.

c. Identify the horizontal asymptote(s), if any exist, of $h(x)$ and justify the existence by writing a limit.

d. Explain why the Intermediate Value Theorem guarantees a value of $c$ on the interval [1.5, 2.5] such that $h(c) = -4$. Then, find $c$.

## AP Calculus Free Response Practice #2
## Calculator NOT Permitted




Graph of $g(x)$

$$f(x) = \begin{cases} ax + 3, & x < -3 \\ x^2 - 3x, & -3 \le x < 2 \\ bx - 5, & x \ge 2 \end{cases}$$

Equation of $f(x)$

Pictured above is the graph of a function $g(x)$ and the equation of a piece-wise defined function $f(x)$. Answer the following questions.

a. Find $\lim\limits_{x \to 1^+} \left[ 2g(x) - f(x) \cdot \cos \pi x \right]$. Show your work applying the properties of limits.

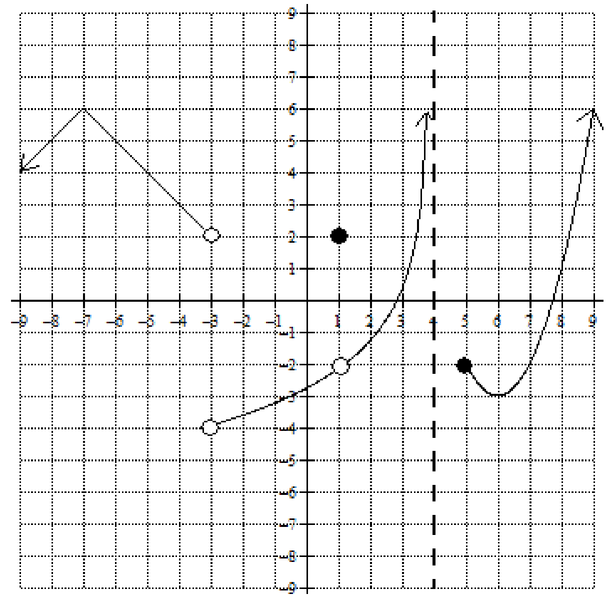
b. On its domain, what is one value of $x$ at which $g(x)$ is discontinuous? Use the three part definition of continuity to explain why $g(x)$ is discontinuous at this value.

c. For what value(s) of $a$ and $b$, if they exist, would the function $f(x)$ be continuous everywhere? Justify your answer.

**AP Calculus**
**Extra Practice on Limits and Multiple Choice Practice**

For questions 1 – 5, refer to the graph of $f(x)$ to the right. Find the value of each indicated limit. If a limit does not exist, give a reason.

| | | |
|---|---|---|
| 1. | $\lim_{x\to -3^+} f(x) + \lim_{x\to 5^+} 3f(x)$ | |
| 2. | $\lim_{x\to 1}\left[\frac{1}{2}f(x)+\cos(\pi x)\right]$ | |
| 3. | $\lim_{x\to 4^-} f(x)$ | |
| 4. | $\lim_{x\to -\infty} f(x)$ | |
| 5. | $\lim_{x\to -3} f(x)$ | |



For questions 6 – 11, find the value of each limit analytically. If a limit does not exist, state why.

6. $\lim_{x\to 0}\dfrac{x^3-2x^2+3x}{x}$

7. $\lim_{x\to 0}\dfrac{3\tan x}{x\sec x}$

8. $\lim_{x\to 3^+}\dfrac{x^2-4}{x^2-x-6}$

9. $\lim_{x\to 2^-}\ln(-x+2)$

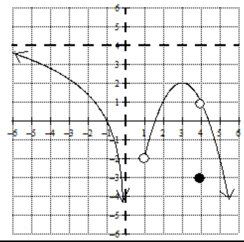
10. $\lim_{x\to\infty}\dfrac{x^3-2x^2+3x}{x^2-3x^3}$

11. $\lim_{x\to\infty} 5+\dfrac{5}{x}$

For question 12 – 16, use the equation $g(x)$ below and the graph of the function $f(x)$.

$$g(x) = \begin{cases} 3|x+3|, & x < -2 \\ \cos\left(\frac{\pi x}{2}\right), & -2 \le x < 2 \\ ax^2 + 2x, & x \ge 2 \end{cases}$$

Graph of $f(x)$



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12. Is $g(x)$ continuous at $x = -2$. [Base your response on the three part definition of continuity.]

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13. For what value(s) of $a$ is $g(x)$ continuous at $x = 2$?

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14. For what value(s) of $b$ is the function $f(x)$ discontinuous? At which of these values does $\lim_{x \to b} f(x)$ exist? Explain your reasoning.

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15. Find $\lim_{x \to 2^+} [g(x) + 2f(x)]$.

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16. Which of the following limits do(es) not exist? Give a reason for your answers.

| $\lim_{x \to 1} f(x)$ | $\lim_{x \to 4} f(x)$ | $\lim_{x \to 0^-} f(x)$ |
|---|---|---|
| | | |

17. Find the values of $k$ and $m$ so that the function below is continuous on the interval $(-\infty, \infty)$.

$$f(x) = \begin{cases} x^2 - kx + 3, & x < -2 \\ 2x - 3, & -2 \le x \le 3 \\ 3 - 2m, & x > 3 \end{cases}$$

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18. $\lim_{x \to 0} \frac{4x-3}{7x+1} =$

A. $\infty$ B. $-\infty$ C. 0 D. $\frac{4}{7}$ E. –3

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19. $\lim_{x \to \frac{1}{3}} \frac{9x^2-1}{3x-1} =$

A. $\infty$ B. $-\infty$ C. 0 D. 2 E. 3

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20. $\lim_{x \to 2} \frac{x^3-8}{x^2-4} =$

A. 4 B. 0 C. 1 D. 3 E. 2

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21. The function $G(x) = \begin{cases} x-3, & x > 2 \\ -5, & x = 2 \\ 3x-7, & x < 2 \end{cases}$ is not continuous at $x = 2$ because…

A. $G(2)$ is not defined B. $\lim_{x \to 2} G(x)$ does not exist C. $\lim_{x \to 2} G(x) \ne G(2)$

D. Only reasons B and C E. All of the above reasons.

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22. $\lim_{x \to \infty} \frac{-3x^2+7x^3+2}{2x^3-3x^2+5} =$

A. $\infty$ B. $-\infty$ C. 1 D. $\frac{7}{2}$ E. $-\frac{3}{2}$

23. $\lim_{x \to -2} \frac{\sqrt{2x+5}-1}{x+2} =$

A. 1

B. 0

C. $\infty$

D. $-\infty$

E. Does Not Exist

24. If $f(x) = 3x^2 - 5x$, then find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

A. $3x - 5$
B. $6x - 5$
C. $6x$
D. 0
E. Does not exist

25. $\lim_{x \to -\infty} \frac{2-5x}{\sqrt{x^2+2}} =$

A. 5

B. –5

C. 0

D. $-\infty$

E. $\infty$

26. The function $f(x) = \frac{2x^2+x-3}{x^2+4x-5}$ has a vertical asymptote at $x = -5$ because…

A. $\lim_{x \to -5^+} f(x) = \infty$

B. $\lim_{x \to -5^-} f(x) = -\infty$

C. $\lim_{x \to -5^-} f(x) = \infty$

D. $\lim_{x \to \infty} f(x) = -5$

E. $f(x)$ does not have a vertical asymptote at $x = -5$

27. Consider the function $H(x) = \begin{cases} 3x-5, & x < 3 \\ x^2 - 2x, & x \ge 3 \end{cases}$. Which of the following statements is/are true?

I. $\lim_{x \to 3^-} H(x) = 4$.

II. $\lim_{x \to 3} H(x)$ exists.

III. $H(x)$ is continuous at $x = 3$.

A. I only

B. II only

C. I and II only

D. I, II and III

E. None of these statements is true