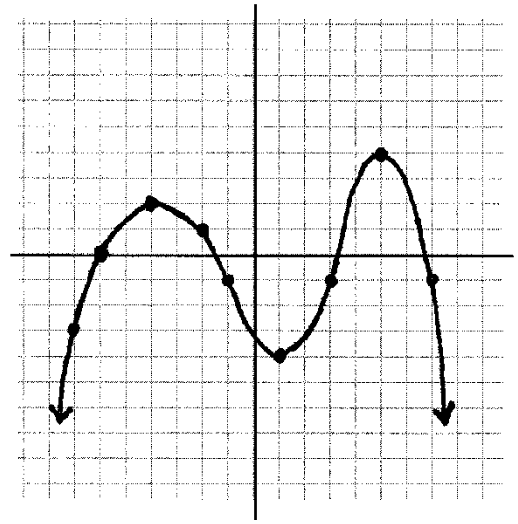
Lesson #2- Understanding the Derivative from a Graphical and Numerical Approach



So far, our understanding of the derivative is that it represents the slope of the tangent line drawn to a curve at a point.

Complete the table below, estimating the value of $f\,'(x)$ at the indicated x – values by drawing a tangent line and estimating its slope.

x – Value	Estimation of Derivative	Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum	Equation of the tangent line at this value of *x*.
–7			
–6			
–4			
–2			
–1			
1			
3			
5			
7			

Based on what you observed in the table on the previous page, what inferences can you make about the value of the derivative, $f'(x)$, and the behavior of the graph of the function, $f(x)$?

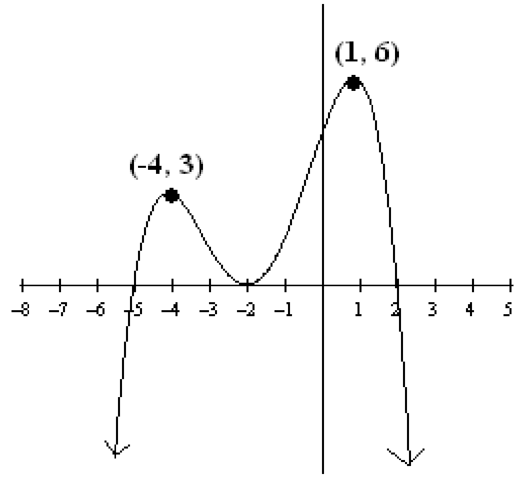
Numerically, the value of the derivative at a point can be estimated by finding the slope of the secant line passing through two points on the graph on either side of the point for which the derivative is being estimated.

x	–3	0	1	4	6	10
***f*(*x*)**	2	1	–3	0	–7	2

***x* – Value**	**Estimation of Derivative**	**Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum**	**Equation of the tangent line at this value of *x*.**
0			
1			
4			
6			

The graph of a function, $g(x)$, is pictured to the right. Identify the following characteristics about the graph of the derivative, $g'(x)$. Give a reason for your answers.

The interval(s) where $g'(x) < 0$	
The interval(s) where $g'(x) > 0$	
The value(s) of x where $g'(x) = 0$	

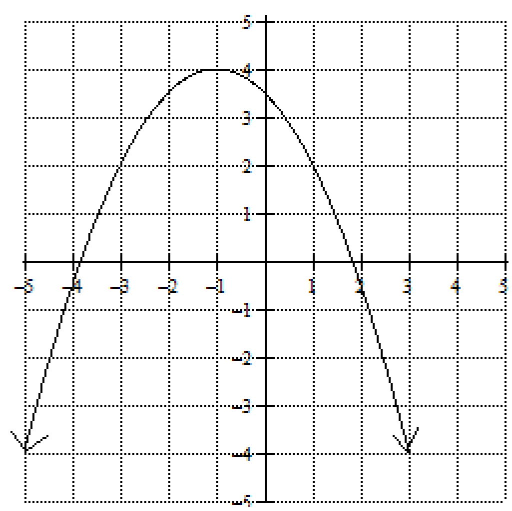


Definition of the Normal Line

Pictured to the right is the graph of $f(x) = -\frac{1}{2}(x+1)^2 + 4$.

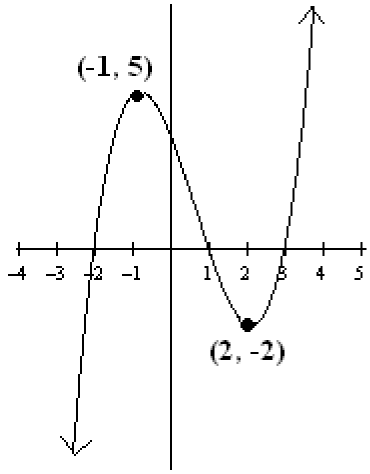
Draw the tangent line to the graph of $f(x)$ at $x = 1$. Then, estimate the value of $f'(1)$.

Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$.



The normal line is the line that is perpendicular to the tangent line at the point of tangency. Draw this line and find the equation of the normal line.

The graph of the derivative, $h'(x)$, of a function $h(x)$ is pictured below. Identify the following characteristics about the graph of $h(x)$ and give a reason for your responses.



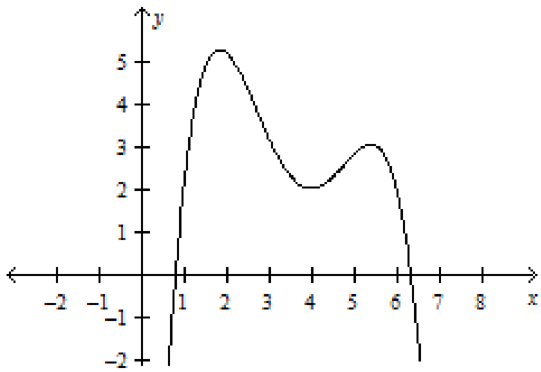
The interval(s) where $h(x)$ is increasing	
The interval(s) where $h(x)$ is decreasing	
The value(s) of x where $h(x)$ has a relative maximum.	
The value(s) of x where $h(x)$ has a relative minimum.	
If $h(-1) = \frac{1}{2}$, what is the equation of the tangent line drawn to the graph of $h(x)$ at $x = -1$?	
If $h(2) = -3$, what is the equation of the normal line drawn to the graph of $h(x)$ at $x = 2$?	

Name____Date___________________Class__________

Lesson #2 Homework

1. The line defined by the equation $2y+3=-\frac{2}{3}(x-3)$ is tangent to the graph of $g(x)$ at $x = -3$. What is the value of $\lim_{x\to -3}\frac{g(x)-g(-3)}{x+3}$? Show your work and explain your reasoning.

Use the graph of $f(x)$ pictured to the right to perform the actions in exercises 2 – 6. Give written explanations for your choices.



2. Label a point, A, on the graph of $y = f(x)$ where the derivative is negative.

3. Label a point, B, on the graph of $y = f(x)$ where the value of the function is negative.

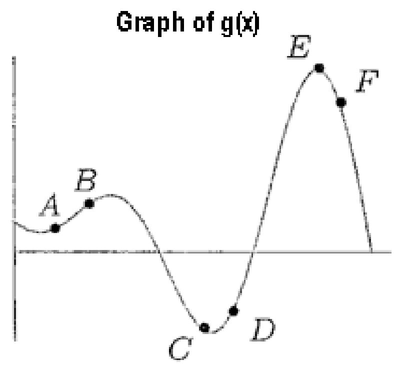
4. Label a point, C, on the graph of $y = f(x)$ where the derivative is greatest in value.

5. Label a point, D, on the graph of $y = f(x)$ where the derivative is zero.

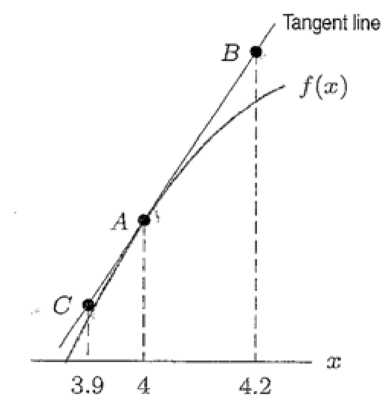
6. Label two different points, E and F, on the graph of $y = f(x)$ where the values of the derivative are opposites.

7. Match the points on the graph of $g(x)$ with the value of $g'(x)$ in the table.

Value of $g'(x)$	Point on $g(x)$
–3	
–1	
0	
½	
1	
2	



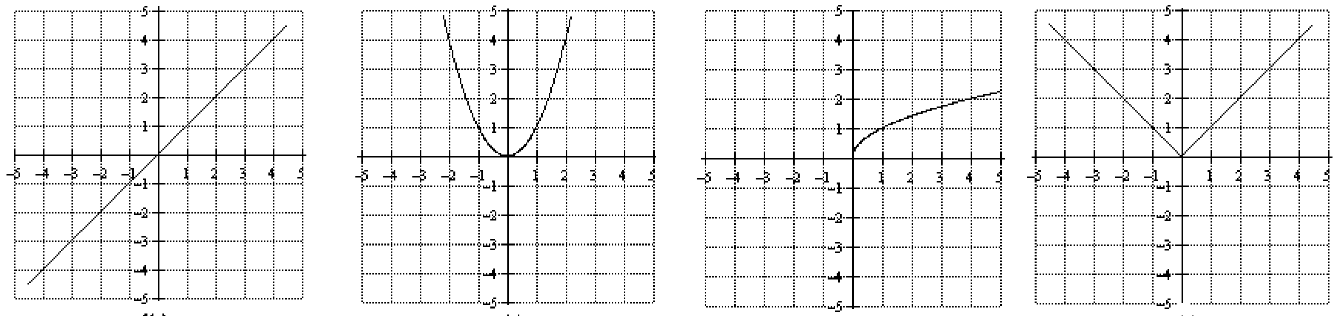
8. The function to the right is such that $f(4) = 25$ and $f'(4) = 1.5$. Find the coordinates of A, B, and C.



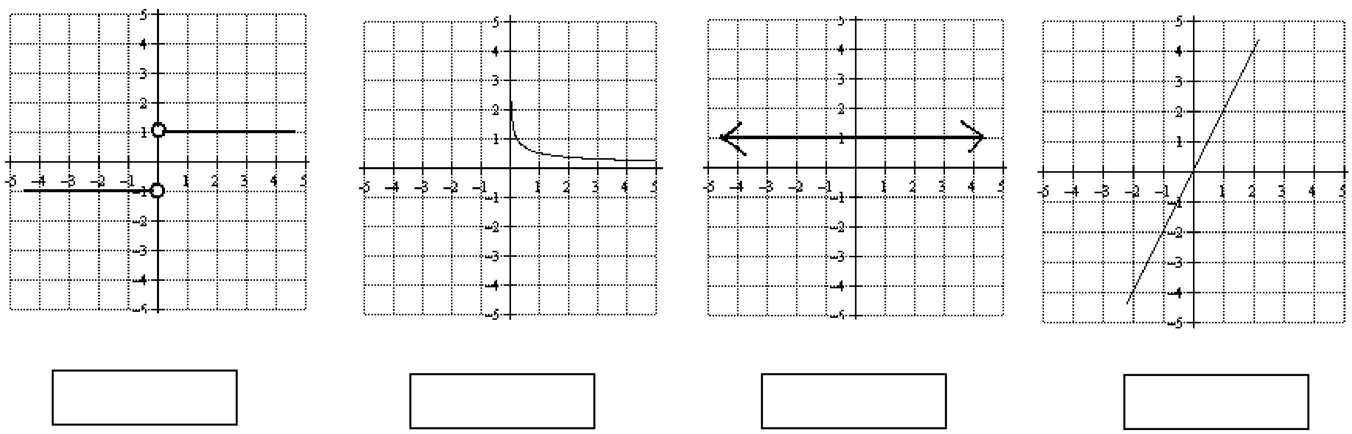
For exercises 9 – 11, use the function $f(x) = \dfrac{1}{x+1}$.

9. Find $f'(x)$.	10. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x = 0$.
	11. Find the equation of the normal line drawn to the graph of $f(x)$ at $x = 0$.

12. Given below are graphs of four functions—$f(x)$, $g(x)$, $h(x)$, and $p(x)$. Below those graphs are graphs of their derivatives. Label the graphs below as $f'(x)$, $g'(x)$, $h'(x)$, and $p'(x)$.



f(x) g(x) h(x) p(x)



The table below represents values on the graph of a cubic polynomial function, $h(x)$. Use the table to complete exercises 13 – 15.

x	–3	–2	–1	0	1	2	4
$h(x)$	–24	0	8	6	0	–4	18

13. Two of the zeros of $h(x)$ are listed in the table. Between which two values of x does the Intermediate Value Theorem guarantee that a third value of x exists such that $h(x) = 0$? Explain your reasoning.

14. Estimate the value of $h'(1.5)$. Based on this value, describe the behavior of $h(x)$ at $x = 1.5$. Justify your reasoning.

15. Estimate the value of $h'(-1.75)$. Based on this value, describe the behavior of $h(x)$ at $x = -1.75$. Justify your reasoning.