Lesson #3- Analytically Finding the Derivative of Polynomial, Polynomial Type, Sine, and Cosine Functions

Consider the function $f(x) = 3$. What does the graph of this function look like? If a tangent line were drawn to $f(x)$ at any value of x, what would the slope of that tangent line be?

Based on this though process, if $f(x) = c$, where c is any constant, then $f\,'(x) =$ ___________ .

Shown below are 6 different polynomial, or polynomial–type, functions. Watch as I find the derivative of each function. See if you can figure out the algorithm that I am using for each function.

Function, $f(x)$	Derivative, $f\,'(x)$
$f(x) = 3x^2 - 2x + 3$	
$f(x) = -5x^3 + 2x^2 - 3x + 1$	
$f(x) = 6 - 3x^3 + 6x^4$	
$f(x) = -2x^{-1} + 3x^{-2}$	
$f(x) = 6x^{\frac{2}{3}} + 4x^{-2}$	
$f(x) = -6x^{-\frac{1}{2}} + 3x^{\frac{1}{2}}$	

Based on what you have seen in the table above, you should now be able to infer how to complete the following Power Rule for Differentiation.

$$\frac{d}{dx}\left[x^n\right] = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

In order to apply the Power Rule for Differentiation, the equation must be written in “polynomial form.” To what do you suppose “polynomial form” refers?

Find $f'(x)$ for each of the following functions. Leave your answers with no negative or rational exponents and as single rational functions, when applicable.

$f(x)=\dfrac{2}{x^2}-4x^3$	$f(x)=\dfrac{3x^4-3x^2-2x}{x}$
$f(x)=(x+3)(x+2)(2x+1)$	$f(x)=\dfrac{x^3-5x^2}{x^5}$
$f(x)=\dfrac{3x}{\sqrt[3]{x^2}}$	$f(x)=-4x^{\frac{3}{4}}+2x^{\frac{1}{4}}$

Remember two trigonometric identities that we will use to find the derivatives of the sine and cosine functions.

$\cos(a + b) =$ ____

$\sin(a + b) =$ ____

Use $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$ for each of the following functions. Your results will show the derivative of the sine and cosine functions.

$f(x) = \sin x$	$f(x) = \cos x$

$\frac{d}{dx}[\sin x] =$ ______________________ $\frac{d}{dx}[\cos x] =$ ______________________

For each of the following functions, find the equation of the tangent line to the graph of the function at the given point.

$f(x) = (2x-1)(x+1)^2$ when $x = -1$	$f(\theta) = 4\sin\theta - \theta$ when $\theta = \frac{\pi}{2}$
$g(\theta) = 2\theta + 3\cos\theta$ when $\theta = \pi$	$h(x) = \dfrac{2x}{\sqrt{x^3}}$ when $x = 2$

Given the equation of a function, how might you determine the value(s) at which the function has a horizontal tangent? Explain your reasoning.

At what value(s) of x will the function $f(x) = x^3 + x$ have a horizontal tangent?

At what value(s) of θ at which the function $f(\theta) = \theta + \sin\theta$ has a horizontal tangent on the interval $[0, 2\pi)$?

Name____**Date**_____________________**Class**__________

Lesson #3 Homework

For exercises 1 – 12, find the derivative of each function. Leave your answers with no negative or rational exponents and as single rational functions, when applicable.

1. $f(x) = 5 - 2x^2 - 3x^3$	2. $h(x) = \dfrac{2x^3 + 3x^2 - 2x}{x}$
3. $h(x) = \dfrac{3}{x^7}$	4. $g(x) = \dfrac{2x^5}{x^8}$
5. $f(\theta) = -3\theta^2 - \cos\theta$	6. $h(x) = \sqrt[3]{x^2}$
7. $g(\theta) = \sqrt{\theta} + 2\sin\theta$	8. $p(x) = -2x^{\frac{3}{2}} + \sqrt{x}$

9. $g(x)=(x+3)(2x-1)^2$	10. $h(x)=\dfrac{x^2+2x-2}{x^3}$
11. $f(x)=\dfrac{3x}{\sqrt[3]{x}}$	12. $h(x)=6\sqrt{x}-3\cos x$

13. For what value(s) of x will the slope of the tangent line to the graph of $h(x)=4\sqrt{x}$ be –2? Find the equation of the line tangent to $h(x)$ at this/these x – values. Show your work.

14. Find the equation of the line tangent to the graph of $g(x) = \dfrac{2}{\sqrt[4]{x^3}}$ when $x = 1$.

15. The line defined by the equation $\frac{1}{2}x + 3 = -2(y - 3)$ is the line tangent to the graph of a function $f(x)$ when $x = a$. What is the value of $f\,'(a)$? Show your work and explain your reasoning.

16. The line defined by the equation $y - 3 = -\frac{2}{3}(x + 3)$ is the line tangent to the graph of a function $f(x)$ at the point (–3, 3). What is the equation of the normal line when $x = –3$. Explain your reasoning.

17. Determine the value(s) of x at which the function $f(x) = x^4 - 8x^2 + 2$ has a horizontal tangent.

18. Determine the value(s) of θ at which the function $f(\theta) = \sqrt{3}\theta + 2\cos\theta$ has a horizontal tangent on the interval $[0, 2\pi)$.

19. For what value(s) of k is the line $y = 4x - 9$ tangent to the graph of $f(x) = x^2 - kx$?