Lesson \#4- Connections between $F(x)$ and $F^{\prime}(x)$ for Polynomial and Trigonometric Functions

| $\boldsymbol{F} \boldsymbol{\prime}(\boldsymbol{x})$ | $\boldsymbol{F}(\boldsymbol{x})$ |
| :---: | :---: |
| Is $=0$ |  |
| Is $>0$ |  |
| Is <0 |  |
| Changes from positive to negative |  |
| Changes from negative to positive |  |




Graph of $f(x)$

Pictured below is the graph of a function $f(x)$. Answer the following questions about the derivative.

1. Approximate the value of $f^{\prime}(4)$.
2. At what values) of $x$ is $f^{\prime}(x)=0$. Justify your answer.


3. On what open interval(s) is $f^{\prime}(x)<0$ ? Justify your answer.
4. On what open interval(s) is $f^{\prime}(x)>0$ ? Justify your answer.
5. At what values) of $x$ does the graph of $f^{\prime}(x)$ go from being below the $x$-axis to above the $x$-axis? Justify your answer.
6. At what values) of $x$ does the graph of $f^{\prime}(x)$ go from being above the $x$-axis to below the $x$-axis? Justify your answer.

Pictured below is the graph of $f^{\prime}(x)$ on the interval $[-3,4]$. Answer the following questions about $f(x)$.

1. On what open intervals) is the graph of $f(x)$ increasing? Justify your reasoning.

2. On what open intervals) is the graph of $f(x)$ decreasing? Justify your answer.

3. At what values) of $x$ does the graph of $f(x)$ have a horizontal tangent? Justify your answer.
4. At what values) of $x$ does the graph of $f(x)$ have a relative maximum? Justify your answer.

5. At what values) of $x$ does the graph of $f(x)$ have a relative minimum? Justify your answer.
6. What is the slope of the tangent line to the graph of $f(x)$ at $x=0$ ? Justify your reasoning.
7. What is the slope of the normal line to the graph of $f(x)$ at $x=4$ ? Justify your reasoning.

For each of the given functions, determine the interval(s) on which $f(x)$ is increasing and/or decreasing. Find all coordinates of the relative extrema. Unless otherwise noted, perform the analysis on all values on $(-\infty, \infty)$. Provide justification for your answers.

1. $f(x)=x^{3}-6 x+1$
2. $f(x)=3 x^{5}-5 x^{3}$

3. $f(\theta)=\theta+2 \sin \theta$ on $(0,2 \pi)$

$\qquad$

## Lesson \#4 Homework

For exercises $1-3$, determine on what intervals the given function is increasing or decreasing. Also, identify the coordinates of any relative extrema of the function. Show your work and justify your reasoning.

1. $f(x)=2 x^{3}+3 x^{2}-12 x$
2. $g(x)=x^{3}-6 x^{2}+15$
3. $h(x)=(x+2)^{2}(x-1)$
4. Pictured to the right is the graph of $f^{\prime}(x)$. On what interval(s) is the graph of $f(x)$ increasing or decreasing? Justify your reasoning.

5. Pictured to the right is the graph of $f^{\prime}(x)$. At what value(s) of $x$ does the graph of $f(x)$ have a relative maximum/minimum? Justify your reasoning.

