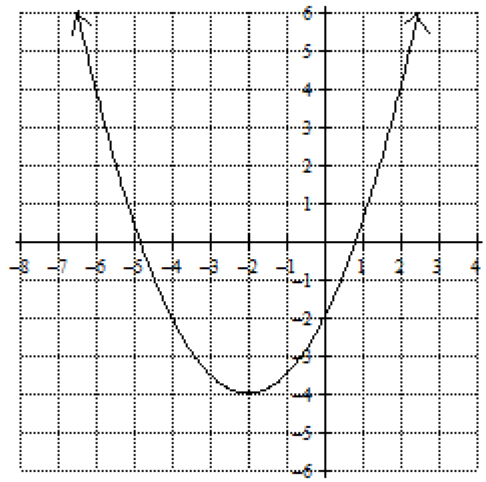
Lesson #5- Solidifying the Concept of the Derivative as the Tangent Line

Pictured to the right is the graph of a quadratic function, $g(x) = \frac{1}{2}(x+2)^2 - 4$.



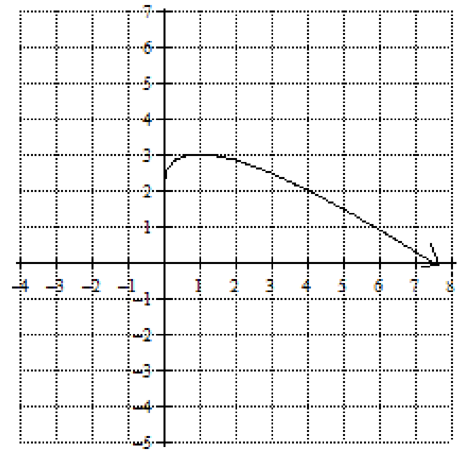
1. Find $g'(-4)$ and explain what this value represents in terms of the graph of the function $g(x)$.

2. Find the equation of the tangent line drawn to the graph of $g(x)$ at $x = -4$. Sketch a graph of this tangent line on the grid with the graph of $g(x)$ above.

3. Using the equation of the tangent line, find the value of y when $x = -3.9$. Then, find the value of $g(-3.9)$.

4. What do you notice about the values of these two results from question 3? What does this imply about how the equation of the tangent line might be used?

Pictured to the right is the graph of the function $g(x) = 2\sqrt{x} - x + 2$. Use the graph and the equation to answer questions 5 – 9.



5. Based on the graph, at what value(s) does the graph of $g(x)$ have a horizontal tangent? Give a reason. Show an algebraic analysis that supports your answer.

6. On what interval(s) is $g'(x) < 0$? Give a reason for your answer.

7. On what interval(s) is $g'(x) > 0$? Give a reason for your answer.

8. For what value(s) of x is the slope of the tangent line equal to 2? Show your work.

9. Find an equation of the tangent line drawn to the graph of $g(x)$ when $x = 4$. Then, draw the tangent line on the grid above.

The table of values below represents values on the graph of the derivative, $h'(x)$, of a polynomial function $h(x)$. The zeros indicated in the table are the only zeros of the graph of $h'(x)$. Use the table to answer questions 10 – 15.

x	−8	−5	−2	0	3	5	7	10	12
***h'*(x)**	11	5	0	−1	−3	−1	0	−3	−9

10. On what interval(s) is the function $h(x)$ increasing and decreasing? Give reasons for your answers.

11. At what x – value(s) does the graph of $h(x)$ have a relative maximum? Justify your answer.

12. At what x – value(s) does the graph of $h(x)$ have a relative minimum? Justify your answer.

13. If $h(3) = 2$, what is the equation of the tangent line to the graph of $h(x)$ at $x = 3$? What is the equation of the normal line to the graph of $h(x)$ at $x = 3$?

14. Find the tangent line approximation of $h(3.1)$.

15. Find the value of each of the following limits:

$$\lim_{x \to -\infty} h(x) \qquad\qquad \lim_{x \to \infty} h(x)$$

The derivative of a polynomial function, $f(x)$, is given by the equation $f'(x) = x(2-x)(x+3)$. Use this equation to answer questions 16 – 20.

16. On what intervals is $f(x)$ increasing? Decreasing? Justify your answers.

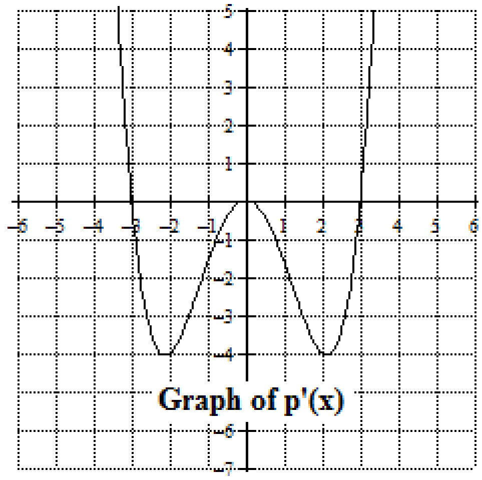
17. At what value(s) of x does the graph of $f(x)$ reach a relative minimum? Justify your answers.

18. At what value(s) of x does the graph of $f(x)$ reach a relative maximum? Justify your answers.

19. If $f(4) = -1$, what is the equation of the tangent line drawn to the graph of $f(x)$ at $x = 4$?

20. Approximate the value of $f(4.1)$. Explain why this is a good approximation of the true value of $f(4.1)$.

Pictured to the right is a graph of $p'(x)$, the derivative of a polynomial function, $p(x)$. Use the graph to answer the questions 21 – 25.



21. On what interval(s) is the graph of $p(x)$ decreasing? Justify your answer.

22. On what interval(s) is the graph of $p(x)$ increasing? Justify your answer.

23. At what value(s) of x does the graph of $p(x)$ reach a relative maximum? Justify your answer.

24. At what value(s) of x does the graph of $p(x)$ reach a relative minimum? Justify your answer.

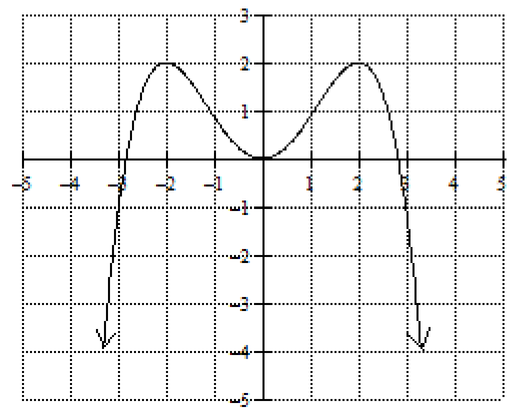
25. Approximate the value of $p(1.8)$ using the tangent line approximation if $p(2) = -3$.

Name____Date____________________Class__________

Lesson #5 Homework

1. If $g'(x) = (x-3)^2(x+1)$, determine on what intervals the graph of $g(x)$ is increasing or decreasing and identify the value(s) of x at which $g(x)$ has a relative maximum or minimum. Justify your reasoning and show your work.

For exercises 2 – 4, use the graph of t function, $h(x)$, pictured to the right. Use the graph to identify the following. **<u>Provide written justification</u>.**

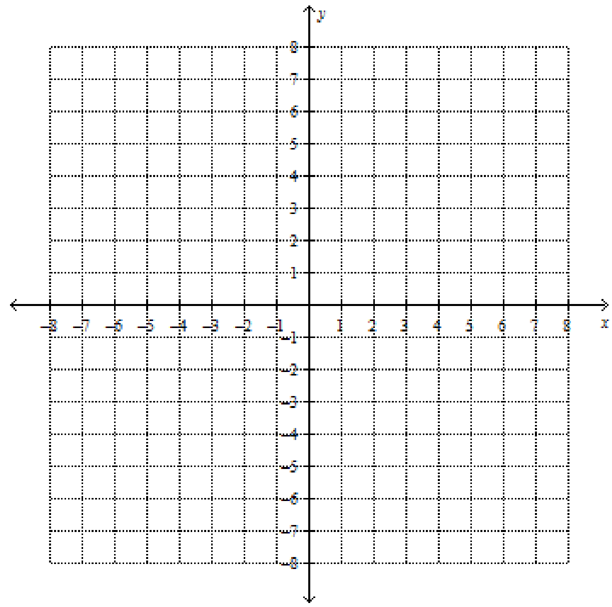


2. On what interval(s) is $h'(x) < 0$?

3. On what interval(s) is $h'(x) > 0$?

4. At what value(s) of x does $h'(x)$ change from positive to negative? From negative to positive?

Consider the quadratic function $f(x) = -\frac{1}{2}x^2 - x + 4$.



5. Sketch an accurate graph of the function.

6. Find $f'(x)$ and use it to find the absolute maximum of the graph of $f(x)$.

7. Estimate the value of $f'(0)$ and explain what this value represents in terms of the graph of $f(x)$.

8. Find the equation of the tangent line to the graph of $f(x)$ at $x = 0$. Draw a graph of this line.

9. Sketch a graph of the normal line to the tangent line at $x = 0$. What is the equation of this line?

10. Use the equation of the tangent line to approximate $f(0.1)$. Then, find $f(0.1)$ using the equation of $f(x)$. Is the approximation an under or over approximation of the actual value of $f(0.1)$? Based on the graph of $f(x)$, why do you suppose this is true?

11. For what function does $\lim_{h\to 0} \frac{2\sin(x+h)-2\sin x}{h}$ give the derivative? Find the limit.

12. Find $\lim_{h\to 0} \frac{(x+h)^5 - x^5}{h}$.

13. Find $\lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h}$.

14. If $f(x) = \frac{3x}{\sqrt{x}}$, what is the slope of the normal line to the graph of $f(x)$ when $x = 4$?

15. If $2x - 3 = 5(y + 1)$ is the equation of the normal line to the graph of $f(x)$ when $x = a$, find the value of $f'(a)$. Show your work and explain your reasoning.

16. On the interval [0, 2π), find the coordinates of the relative minimum(s) of $f(\theta) = \sqrt{3}\theta - 2\sin\theta$.

The derivative of a function $f(x)$ is $f'(x) = (3-x)^2(x+5)$. Use this derivative for exercises 17 and 18.

17. At what value(s) of x does the graph of $f(x)$ have a relative maximum? Justify your answer.

18. Use the equation of the tangent line to approximate the value of $f(2.1)$ if $f(2) = -3$.